

WIND STRESSES IN TALL BUILDINGS

A Comparative Analysis

G. M. Jones

Ann Arbor, Michigan

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By Successive Approximations and Six Approximate Methods.

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This paper is taken from a dissertation submitted by the writer to the Univ. of Michigan in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering.

WIND STRESSES IN TALL BUILDINGS

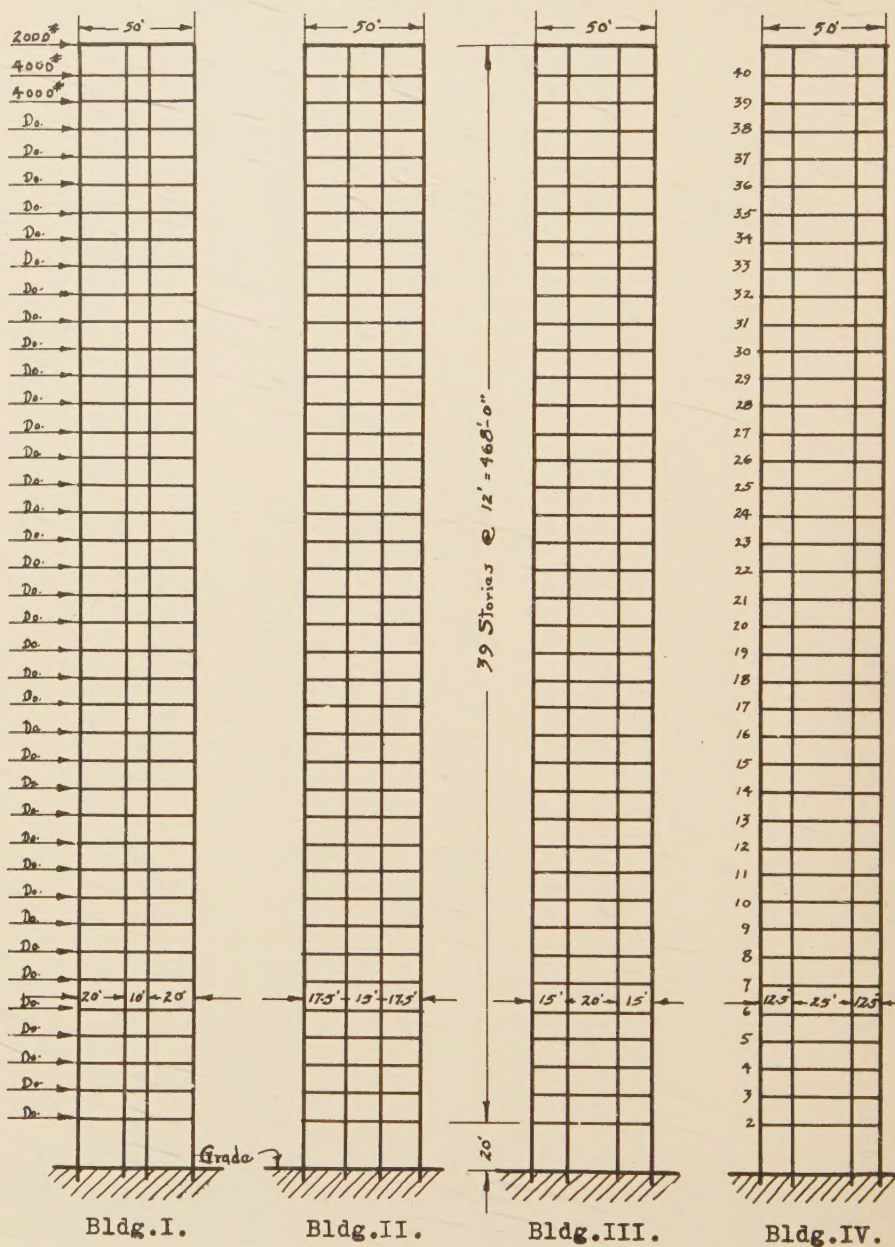


Fig.1. - STEEL FRAMES--Scale 3/16" = 1'--0".



INTRODUCTION

In the practical analysis of the wind stresses in tall buildings designers have resorted to several different approximate methods with no knowledge of the degree of error that exists in using the particular method in the design of a particular building. Two buildings with different base-height ratios may require two different methods to approximate their wind stresses. The distribution of columns, nearer or farther away from the center line of a building should affect the method used in analysis.

If various approximate methods are each used for the analysis of a particular building and some so-called accurate solution is applied to the same building, relative values of the various approximate methods can be ascertained for this particular building. Now, if this building goes through a series of systematic changes and the methods, both approximate and accurate, are applied to each change, one can study how the various approximate methods vary with reference to the changes made from the original building. Conclusions may be drawn from the results of such a procedure that will be of value to the designer about to choose a method of analysis.

In the work that follows four symmetrical building frames

are investigated (Fig. 1), each 40 stories in height with a width of 50 feet, first story, 20 feet high and other 39 at 12 feet each. They are each designed to a constant live load of 50 lbs. per sq. ft. with their proper dead load and a wind load on each building of 4000 lbs. applied at each floor level. Numbering the building frames from I. to IV., number I. has a center bay of 10 feet, number II., a center bay of 15 feet, number III., a center bay of 20 feet and number IV., a center bay of 25 feet. Thus from Building I. to Building IV. the center span tends to increase in equal increments and, of course, the side bays decrease accordingly.

The first analysis of each building will be according to the slope deflection theory and assumptions, made simple by the use of the Hardy Cross Method carried to an accurate degree of approximations. Using these results as a "Standard of Comparison" several of the most common approximate methods will be applied to these same buildings and compared. If the designer's frame is reasonably close to any of the frames analysed here or if it fits into the trend that they determine the results of this work should be directly useful as a guide.

The tabulations of the properties of columns and girders (Tables 1., 2., 3.) are the sections used in the problem,

designed and revised for wind stress according to present day practice.*

STANDARD OF COMPARISON

The Fixed-End Moment Distribution Method** is used for determining the stresses for the STANDARD OF COMPARISON.

Assumptions made are as follows:

1. The joint connections are perfectly rigid.
2. Longitudinal distortions are neglected.
3. Distances are measured from neutral axis to neutral axis.
4. The deflection of a member due to the internal shearing stress is zero.
5. The wind load is resisted entirely by the steel frame (as shown on page 2.).

The steps of procedure are as follows:

1. Calculate the moments in the columns due to the lateral forces, considering the joints fixed against rotation, but free to deflect laterally. The sum of the moments at the top and the bottom of all columns of a story is equal to the shear in the story multiplied by

* Burt, H. J. "Steel Construction", Parts II., III., IV., pages 85-346 (omitting tables). American Technical Society, New York City, 1927.
"Pocket Companion", Carnegie Steel Co., Pittsburgh, Pa., 1927.

** Cross, Hardy, "Analysis of Continuous Frames by Distributing Fixed-End Moments", Proceedings of the American Society of Civil Engineers, Vol. 56, pages 919-928. American Society of Civil Engineers Headquarters, New York City, May 1930.

WIND STRESSES IN TALL BUILDINGS

PROPERTIES OF COLUMNS

Story	Building I.				Building II.			
	Outer Cols.		Inner Cols.		Outer Cols.		Inner Cols.	
	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)
1	160.9	9741.	132.9	7559.	146.4	8859.	146.4	8859.
2	127.4	7254.	122.4	7014.	117.2	6928.	127.4	7254.
3	122.4	7014.	122.4	7014.	113.2	6552.	127.4	7254.
4	122.4	7014.	122.4	7014.	113.2	6552.	127.4	7254.
5	113.2	6552.	113.2	6552.	105.2	5830.	122.4	7014.
6	113.2	6552.	113.2	6552.	105.2	5830.	122.4	7014.
7	105.2	5830.	109.2	6187.	96.2	5457.	113.2	6552.
8	105.2	5830.	109.2	6187.	96.2	5457.	113.2	6552.
9	96.2	5457.	101.2	5484.	88.2	4910.	105.2	5830.
10	96.2	5457.	101.2	5484.	88.2	4910.	105.2	5830.
11	88.2	4910.	96.2	5457.	81.2	4327.	101.2	5484.
12	88.2	4910.	96.2	5457.	81.2	4327.	101.2	5484.
13	81.2	4327.	88.2	4910.	74.2	3776.	92.2	5120.
14	81.2	4327.	88.2	4910.	74.2	3776.	92.2	5120.
15	77.7	4048.	81.2	4327.	70.7	3512.	84.7	4615.
16	77.7	4048.	81.2	4327.	70.7	3512.	84.7	4615.
17	70.7	3512.	74.2	3776.	63.7	3006.	81.2	4327.
18	70.7	3512.	74.2	3776.	63.7	3006.	81.2	4327.
19	63.7	3006.	70.7	3512.	60.2	2764.	74.2	3776.
20	63.7	3006.	70.7	3512.	60.2	2764.	74.2	3776.
21	60.2	2764.	63.7	3006.	53.2	2302.	67.2	3255.
22	60.2	2764.	63.7	3006.	53.2	2302.	67.2	3255.
23	53.2	2302.	56.7	2529.	47.9	2053.	60.2	2764.
24	53.2	2302.	56.7	2529.	47.9	2053.	60.2	2764.
25	47.9	2053.	53.2	2302.	43.5	1885.	53.2	2302.
26	47.9	2053.	53.2	2302.	43.5	1885.	53.2	2302.
27	41.8	1857.	45.8	1970.	38.3	1643.	47.9	2053.
28	41.8	1857.	45.8	1970.	38.3	1643.	47.9	2053.
29	36.5	1539.	40.0	1749.	34.8	1436.	40.0	1749.
30	36.5	1539.	40.0	1749.	34.8	1436.	40.0	1749.
31	32.5	1351.	34.8	1436.	30.2	1261.	34.8	1436.
32	32.5	1351.	34.8	1436.	30.2	1262.	34.8	1436.
33	27.9	1044.	30.9	1169.6	25.0	921.3	30.9	1169.6
34	27.9	1044.	30.9	1169.6	25.0	921.3	30.9	1169.6
35	20.0	738.8	22.1	823.5	20.0	738.8	22.1	823.5
36	20.0	738.8	22.1	823.5	20.0	738.8	22.1	823.5
37	15.6	552.5	17.1	609.4	14.1	496.0	17.1	609.4
38	15.6	552.5	17.1	609.4	14.1	496.0	17.1	609.4
39	9.7	333.4	9.7	333.4	9.7	333.4	9.7	333.4
40	9.7	333.4	9.7	333.4	9.7	333.4	9.7	333.4

Table 1.

PROPERTIES OF COLUMNS

Story	Building III.				Building IV.			
	Outer Cols.		Inner Cols.		Outer Cols.		Inner Cols.	
	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)	Area sq.in.	I(in ⁴)
1	132.9	7559.	155.9	9248.	113.2	6552.	165.9	10248.
2	109.2	6187.	137.4	7981.	96.2	5457.	146.4	8859.
3	105.2	5830.	132.9	7559.	92.2	5120.	141.9	8415.
4	105.2	5830.	132.9	7559.	92.2	5120.	141.9	8415.
5	96.2	5457.	127.4	7254.	88.2	4910.	132.9	7559.
6	96.2	5457.	127.4	7254.	88.2	4910.	132.9	7559.
7	88.2	4910.	122.4	7014.	81.2	4327.	127.4	7254.
8	88.2	4910.	122.4	7014.	81.2	4327.	127.4	7254.
9	81.2	4327.	113.2	6552.	74.2	3776.	122.4	7014.
10	81.2	4327.	113.2	6552.	74.2	3776.	122.4	7014.
11	74.2	3776.	105.2	5830.	67.2	3255.	113.2	6552.
12	74.2	3776.	105.2	5830.	67.2	3255.	113.2	6552.
13	70.7	3512.	101.2	5484.	63.7	3006.	105.2	5830.
14	70.7	3512.	101.2	5484.	63.7	3006.	105.2	5830.
15	63.7	3006.	92.3	5120.	56.7	2529.	96.2	5457.
16	63.7	3006.	92.3	5120.	56.7	2529.	96.2	5457.
17	60.2	2764.	84.7	4615.	53.2	2302.	92.2	5120.
18	60.2	2764.	84.7	4615.	53.2	2302.	92.2	5120.
19	53.2	2302.	77.7	4048.	47.9	2053.	81.2	4327.
20	53.2	2302.	77.7	4048.	47.9	2053.	81.2	4327.
21	49.7	2081.	70.7	3512.	43.5	1885.	74.2	3776.
22	49.7	2081.	70.7	3512.	43.5	1885.	74.2	3776.
23	43.5	1885.	63.7	3006.	40.0	1749.	67.2	3255.
24	43.5	1885.	63.7	3006.	40.0	1749.	67.2	3255.
25	38.3	1643.	56.7	2529.	34.8	1436.	60.2	2764.
26	38.3	1643.	56.7	2529.	34.8	1436.	60.2	2764.
27	32.5	1351.	49.7	2081.	33.8	1276.	53.2	2302.
28	32.5	1351.	49.7	2081.	33.8	1276.	53.2	2302.
29	30.9	1169.6	43.5	1885.	27.9	1044.	45.7	1970.
30	30.9	1169.6	43.5	1885.	27.9	1044.	45.7	1970.
31	27.9	1044.0	36.5	1539.	25.0	921.3	40.0	1749.
32	27.9	1044.0	36.5	1539.	25.0	921.3	40.0	1749.
33	22.1	823.5	30.2	1261.	20.0	738.8	34.8	1436.
34	22.1	823.5	30.2	1261.	20.0	738.8	34.8	1436.
35	17.9	656.2	25.0	921.3	15.6	552.5	27.9	1044.
36	17.9	656.2	25.0	921.3	15.6	552.5	27.9	1044.
37	14.1	496.0	17.9	656.2	12.4	431.5	20.0	738.8
38	14.1	496.0	17.9	656.2	12.4	431.5	20.0	738.8
39	8.8	292.0	10.6	365.6	8.8	292.0	12.4	431.5
40	8.8	292.0	10.6	365.6	8.8	292.0	12.4	431.5

Table 2.

PROPERTIES OF GIRDERS

(Moments of Inertia)

Fl.	Building I.		Building II.		Building III.		Building IV.	
	Outer	Middle	Outer	Middle	Outer	Middle	Outer	Middle
2	2811.5	1176.8	2301.5	2811.5	1550.3	3669.7	1214.2	4380.
3	2087.2	877.7	1256.5	1550.3	1169.5	2811.5	795.5	3020.5
4	2087.2	877.7	1256.5	1550.3	1169.5	2734.9	795.5	3020.5
5	1599.7	877.7	1176.8	1466.3	1169.5	2371.8	795.5	2940.5
6	1599.7	837.8	1176.8	1466.3	1169.5	2371.8	609.	2940.5
7	1599.7	837.8	1176.8	1466.3	960.8	2371.8	609.	2940.5
8	1501.7	632.1	1176.8	1466.3	877.7	2230.1	609.	2734.9
9	1501.7	632.1	1176.8	1466.3	877.7	2230.1	609.	2734.9
10	1256.5	632.1	1141.8	1263.5	687.2	2087.2	609.	2457.2
11	1256.5	632.1	1141.8	1263.5	687.2	2087.2	508.7	2457.2
12	1256.5	632.1	1117.1	1263.5	687.2	2087.2	508.7	2457.2
13	1216.6	609.	1117.1	1214.2	659.6	1599.7	481.1	2301.5
14	1216.6	609.	1117.1	1214.2	659.6	1599.7	481.1	2301.5
15	1141.8	609.	877.7	1169.5	609.	1466.3	481.1	2159.8
16	1141.8	508.7	877.7	1169.5	609.	1466.3	441.8	2159.8
17	1141.8	508.7	877.7	1169.5	609.	1466.3	441.8	1794.4
18	917.5	441.8	795.5	917.5	609.	1216.6	319.3	1794.4
19	917.5	441.8	795.5	917.5	481.1	1216.6	319.3	1794.4
20	837.8	441.8	609.	659.6	453.6	1169.5	319.3	1542.9
21	837.8	319.3	609.	659.6	453.6	1169.5	284.1	1542.9
22	837.8	319.3	609.	659.6	453.6	1169.5	284.1	1403.3
23	737.1	284.1	609.	609.	405.5	917.5	246.3	1403.3
24	737.1	284.1	508.7	609.	405.5	917.5	246.3	1214.2
25	609.	284.1	481.1	481.1	284.1	795.5	246.3	1214.2
26	609.	268.9	481.1	481.1	284.1	795.5	246.3	1214.2
27	609.	268.9	481.1	481.1	284.1	609.	199.4	1214.2
28	609.	199.4	481.1	441.8	268.9	609.	199.4	1214.2
29	609.	199.4	481.1	441.8	268.9	609.	199.4	1214.2
30	609.	199.4	481.1	284.1	227.	609.	133.5	1214.2
31	609.	133.5	481.1	284.1	227.	609.	133.5	1214.2
32	609.	133.5	481.1	284.1	227.	609.	133.5	1214.2
33	609.	133.5	481.1	227.	227.	609.	133.5	1214.2
34	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
35	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
36	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
37	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
38	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
39	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
40	609.	91.4	481.1	227.	227.	609.	133.5	1214.2
Rf.	609.	91.4	481.1	227.	227.	609.	133.5	1214.2

Table 3.

the story height and, as the deflections of the columns in a story due to lateral forces are equal, the column moments and shears are proportional to the I/L^2 values of the columns.

2. In step number two, moments are distributed to the members at a joint according to their relative "stiffness". The elastic coefficient for any member or its stiffness, E constant, is proportional to the moment of inertia (I), divided by the length (L).

3. Carry over the distributed moments using a carry-over factor of one half. If one end of a member which is on unyielding supports at both ends is rotated while the other end is held fixed, the ratio of the moment at the fixed end to the moment producing rotation at the rotating end is the "carry-over factor".

4. Sum up the moments at the top and the bottom of the columns and supply the moment, plus or minus, necessary to make it equal the shear in the story times the story height. This moment must be distributed to the different columns according to their moment of inertia.

The entire set of moments from all steps is summed up to be operated upon in the next step (step number 2.) which is the beginning of approximation number two. These steps are continued until the desired accuracy is obtained. These sums may later be used to plot a curve to determine

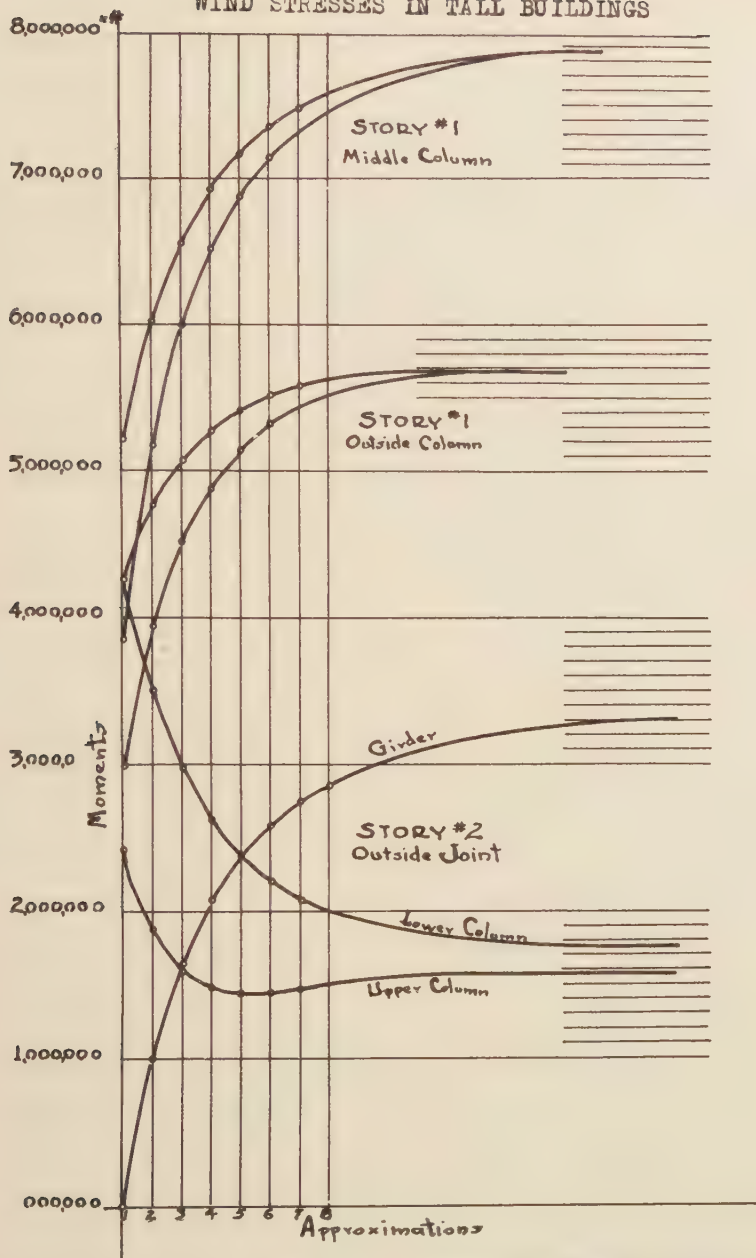


Fig. 3. - MOMENT CURVES. Bldg.III.

WIND STRESSES IN TALL BUILDINGS

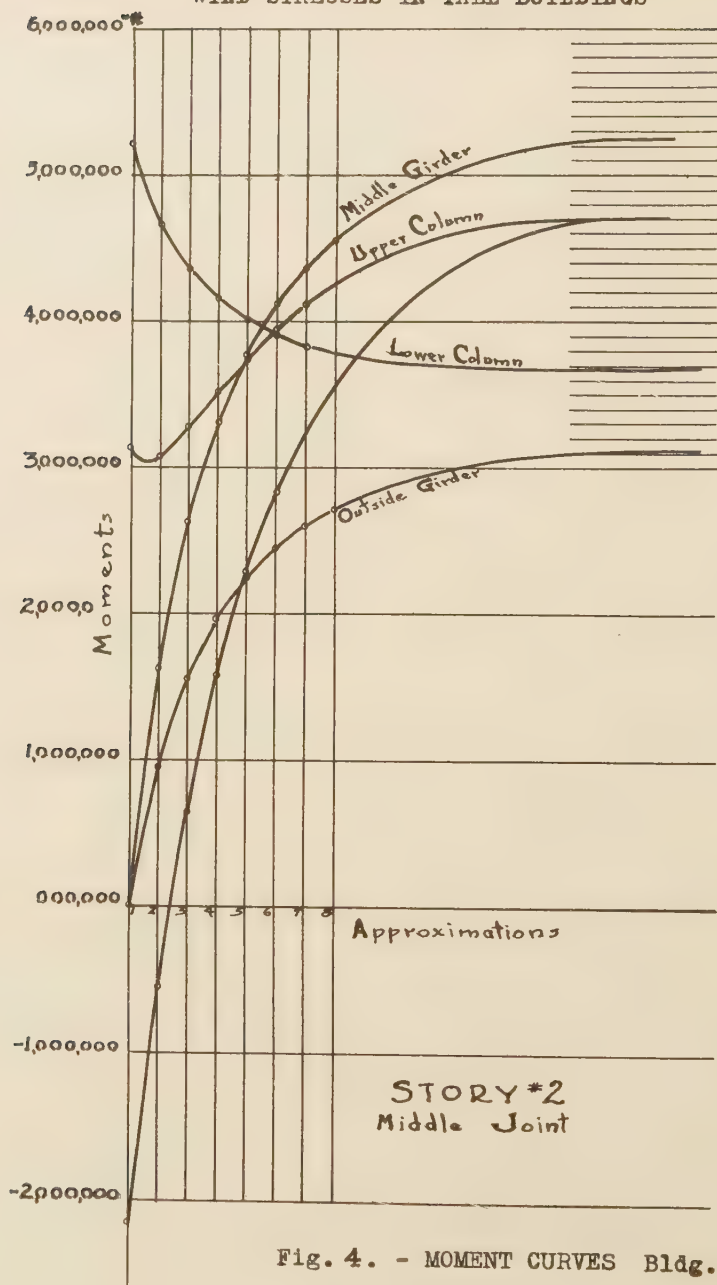


Fig. 4. - MOMENT CURVES Bldg.III.

the convergence and the results.

Figure 2. shows computations of first and second stories of one of the buildings but it is typical of all stories of the four buildings. There are certain values that should be explained (Fig. 2) since they remain constant throughout the series of operations and are listed for convenience and to save time. The eight place decimals, inclined to 45 degrees near each column, are the decimal parts of one-fourth of the shear in the story times the story height that is given to each end of the respective columns. The decimals on each girder and column at the joints are the relative stiffnesses of these members at the joint. The values in the circles are the sums of the I/L values for the members about the joint. The moments in the girders are written parallel to the girders; those in the columns, parallel to the columns. They are arranged to permit direct addition and subtraction, thus upper column moments and moments of the inner end of outside girders start away from the members and are filled in closer to them as the approximations increase. The extra column of figures to the right of the first set are results listed for convenience and for future use in the problem.

The signs used are according to the rule, any moment which tends to rotate a joint clockwise is considered

positive and the reverse negative.

It will be noted that from the values listed at the lower end of the second story outside column (Fig. 2) (as in case of any column listed) there may be five curves traced--the moment value against the cycle number of the approximation. A study of such curves (Fig. 3) shows definite relations to the results sought. The set that starts with underscoring (Fig. 2) tends to determine a smooth curve that starts with a value 2,421,000 then 1,875,447 and then 1,595,463, on down the tabulation. As the number of cycles approaches the answer, the changing in moments, the giving and taking by joints and members approach zero, thus the curve at that point must have a zero tangent. For any joint all the members tend to change values until they are all about balanced. This argument holds for members of joints in the vicinity, thus if any curve lends itself to determining the horizontal length of a curve (the number of approximations), the curves in that section of the building tend to do some changing about (receiving and giving up stress) until they all come to a standstill.

At the first floor outside column joint, the underscoring values are plotted against the number of the approximation to give the upper curve. Since it was a little difficult to determine just where the curve would flatten to horizontal, another set of values, those tabulated just

above the other set, was plotted (Fig. 3). These determine a curve that also approaches the final moment. Having two curves that will both be horizontal and simultaneous at the result gives special aid in plotting curves that will give good results. At the middle joint of story number two (Fig. 4) the second story upper column shows the curves of these two sets of values and how they must meet on the horizontal.

The girder approximations determine smooth curves that also come to balance in the vicinity where the column curves flatten horizontally (zero tangent).

Seven approximations were necessary to get a curve that could be plotted satisfactorily near the bottom of the building and four or five near the top. Between the stories of three or four and about story twenty plotting curves from the approximations would have been more difficult but for the trend of the number of approximations developed (at answer) in plotting from the bottom up to this zone and from the top down to this zone and on into the zone. This fact, number of approximations determined in advance, is additional to that of the simultaneous plotting and the zero tangent aid.

Throughout the problem the number of approximations for good convergence seems large. This is because the building was first designed for live and dead loads and only checked

and revised for wind loads. The columns receive most of their load as direct stress while the girders receive most of theirs through bending. The column moments of inertia for buildings of approximately the heights herein studied have little relation to the wind loads direct. The buildings were designed for dead load, live load and wind load and the stress analysis made for wind stress alone. The dead loads and live loads have the effect of making some members much over size for wind analysis while other members are not affected seriously. In other words the design, being practical, is not balanced for the analysis and the relative stiffnesses of the members at the joints vary largely because of dead and live loads rather than because of wind load. If such structures were designed and analysed for the same loads, the approximations would converge more quickly. This strictly indeterminate method of procedure is not according to practice and would result in large joints for the large resulting moments instead of relatively smaller joints to care for just wind moment.

The difference in the relative stiffnesses of girders to columns in the lower and upper stories is responsible for the difference in the number of approximations necessary for convergence in the lower and upper stories: A girder or column of a joint resists stress in each cycle according to its relative ability to do so. If it is

relatively weak, as girders of the lower stories are, more cycles will be necessary than in a relatively stronger upper girder. Ultimately summation M about the joints must equal zero or there must be as much wind stress in the girders of a joint as there is in the columns.

APPROXIMATE METHODS COMPARED

Fleming's methods I., II., IIA. and III.* are used as a guide throughout these approximations with certain variations considered. These methods are statically determinate.

General assumptions that are adhered to throughout this group of approximations are as follows:

1. The point of contraflexure of each column is at mid-height of the story.
2. All girders of the same floor have the same moment of inertia.
3. The joints are perfectly rigid.
4. Longitudinal distortions are neglected.
5. The wind load is resisted entirely by the frame.
6. Distances are measured from center line to center line.

The following additional assumptions apply for each

* Fleming, Robins, "Six Monographs on Wind Stresses", pages 61-76. Engineering News, Hill Building, New York City, 1915.

method as listed but do not apply to the entire group.

1. CANTILEVER METHOD "A". (Column areas in story assumed equal).

1a. A bent of a frame acts as a cantilever.

1b. The point of contraflexure of each girder is at its mid-length.

1c. All columns in a given story have the same sectional area.

1d. The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent.

2. CANTILEVER METHOD "B". (Column area variations in story considered).

2a. A bent of a frame acts as a cantilever.

2b. The point of contraflexure of each girder is at its mid-length.

2c. The direct stress in the column will vary both as its distance from the neutral axis and its sectional area.

3. EQUAL SHEARS METHOD*

3a. The wind loading taken by each bay is proportional to its span length.

3b. The horizontal shear on any plane is

* Equal shear to each column point of contraflexure was discontinued in favor of this method.

distributed among the columns proportional to their moments of inertia.

4. PORTAL METHOD "A". (Column moments of inertia in story assumed equal).

4a. A bent of a frame acts as a series of independent portals.

4b. The point of contraflexure of each girder is at its mid-length.

4c. The wind loading taken by each bay is proportional to its span length.

4d. The horizontal shear is distributed to each portal in proportion to its span length.

4e. All columns of the same story have the same moment of inertia.

5. PORTAL METHOD "B". (Column moment of inertia variations in story considered).

5a. A bent of a frame acts as a series of independent portals.

5b. The wind loading taken by each bay is proportional to its span length.

5c. The horizontal shear is distributed to each portal in proportion to its span.

5d. Horizontal shear in each portal is distributed to each column in proportion to its moment of inertia.

6. CONTINUOUS PORTAL METHOD.

6a. The direct stress in the column will vary both as its distance from the neutral axis and its sectional area.

6b. The horizontal shear on any plane is distributed among the columns proportional to their moments of inertia.

RECENTLY PROPOSED METHODS

By way of comparison the methods presented by Grinter,* by Goldberg** and by Maugh*** were applied for the 33rd story of Building II. Grinter deflects the structure, calculates fixed-end moments, balances moments, determines shears, applies equal and opposite joint forces causing new deflections, etc. Goldberg, starting approximations with the assumption that points of contraflexure are at mid-lengths of members and that all joints of a story have the same rotation, calculates a first set of R values. He

* Grinter, L. E. Wind Stress Analysis Simplified, Proceedings of the American Society of Civil Engineers, pages 3-27. American Society of Civil Engineers Headquarters, New York City, January 1933.

** Goldberg, J. E. Wind Stresses by Slope Deflection and Converging Approximations, Proceedings of the American Society of Civil Engineers, pages 763-776. American Society of Civil Engineers Headquarters, New York City, May 1933.

*** Maugh, L. C. An analysis of wind bents as developed by Prof. Maugh at Ann Arbor.

then separates the joints alternating in two groups and computes rotation in the second group from values of the first group, and then in the first group from values of the second group. Then a new value of R from which new values for rotation are found, etc. Finally these converged values of R and θ are substituted into the slope deflection equation for end moments. Maugh treats each bay individually by hinging the adjoining members from adjacent bays at their connecting points. Distributing the shear for constant horizontal deflection of each bay in the story and treating the bent as a rectangular frame moments are found. The effect of the column moments of the adjoining stories is brought over into the bent and added to the shear moment for total moment.

Results converged very rapidly in the Grinter Method and the Goldberg Method thus fewer approximations. In the Maugh Method the error is small because the most important carry-over moments coming into the story are from adjacent stories above and below and have a balancing effect on each other.

The following table shows these comparative results in thousands of inch pounds.

COMPARISON OF MOMENTS

Building II.

1000"# Units

33d Story Columns

	Outside Column		Middle Column		Remarks
	Top	Bottom	Top	Bottom	
Cross	450	390	680	630	Machine (7Approx.)
Grinter	435	411	685	630	Slide Rule (4 ")
Goldberg	473	378	707	597	Slide Rule (2 ")
Maugh	434	378	719	628	Slide Rule

DISCUSSION

In a study of assumptions and results obtained it will be noted that the middle girder of the Continuous Portal Method has the same moment as that of the Cantilever Method "B". The column moments of the Continuous Portal Method are the same as for the Method of Equal Shears. The moments for the middle girder of the Equal Shears Method, Portal Method "A" and Portal Method "B" are the same. The Continuous Portal Method and the Method of Equal Shears have the same outer end girder moments in the outside girder.

An assumption that runs through the entire set of approximate methods is that "the point of contraflexure of

each column is at mid-height of the story". Ordinarily this holds true for the columns of these buildings but it is very inaccurate at the first story where the story height is quite different from the ones above. Some of this inaccuracy tends to run into adjoining stories (particularly in the second). The approximate methods used are all on the unsafe side for maximum moments in the design of the first and second story columns. The point of contraflexure in the first story is much higher than mid-height giving a large lower moment and a small upper moment. In the stories near the top of the buildings it is also inaccurate. The lower column moments are smaller and the upper column moments are larger thus the points of contraflexure are lower than mid-point.

The assumption, "the point of contraflexure of each girder is at its mid-length" is accurate regardless of the span variations of the building bents.

If the extra high first story is treated separately an accurate rule is: "The point of contraflexure should be, in the lower stories (starting with the second floor), six-tenths of the height up from the bottom of the column, and in the upper stories (starting with the roof), six-tenths of the height down from the top of the column. The middle half of the height of the building should use column contraflexure at mid-heights of the columns. The mid-points

of all girders should be used as points of contraflexure."

A general assumption that all girders of the same floor have the same moment of inertia seems to have some ill effect on the results since in Buildings III. and IV. this is least true and results are least accurate. In these two buildings there approaches a more equal distribution between large and small moments of inertia.

The girders of the approximate methods have the common property of going from the safe side of the STANDARD OF COMPARISON in the lower stories to the unsafe side in the upper stories or the reverse. This takes place more or less uniformly. It may also be noted that if an inner column is on the safe side of the Standard of Comparison the corresponding outer column is on the unsafe side, or the reverse may be true. This is because the sum of the moments at the top and the bottom of the columns of a story must equal the shear in the story times the story height. The shear in the story is constant for any story and the story height is constant, therefore, their product is a constant value. If one is smaller, another or others must be larger. Also when an outer girder is on the safe side, the corresponding middle girder is on the unsafe side or the reverse.

As the outside bay spans decrease the Continuous Portal Method shows the inner moments for the girder of this span to decrease until in the case of Building IV. many of

the points of contraflexure fall outside of the girders and the method fails. The Method of Equal Shears has such a tendency.

It should be remembered, of course, that a member is designed by its maximum moment and when one end is on the unsafe side the other end may be too conservative resulting in an uneconomical design rather than an unsafe one.

Figures 5, 6, 7 and 8 show graphically the bending moments of all methods of this study. For lack of space the moments for only about half the stories are plotted on the diagrams in four groups of five stories each. The moment diagrams for the middle columns have been drawn opposite those of the outside columns. This is done to permit a minimum overlapping.

The Standard of Comparison is plotted in solid lines. The moments for the approximate methods have broken lines with dots indicating them as follows:

Cantilever Method "A"	-----	one dot
Cantilever Method "B"	-----	two dots
Equal Shears Method	-----	three dots
Portal Method "A"	-----	four dots
Portal Method "B"	-----	five dots
Continuous Portal Method	--	six dots

Where two or more methods coincide lines are sometimes extended to indicate the methods except in the case of the Standard of Comparison (solid line) which is used for any method that coincides with it or nearly so.

CONCLUSION

From the foregoing discussion it is safe to conclude that for three bay buildings, the type presented in this study, any one of the methods may be used for buildings with proportionately small center bays on to bays of about equal spans as a limit. Beyond this, the Method of Equal Shears and the Continuous Portal Method fail in application.

For design purposes all points of contraflexure in all columns and girders (excepting the high first story columns) may be at their mid-points.

A serious error is not made in assuming the column sections and moments of inertia equal. This assumption gives practically as good results as otherwise especially for buildings with proportionately small center bays on to bays of about equal spans as a limit. The Portal Method, column moments of inertia for any story assumed equal, while not entirely satisfactory, gives fair results.

Where the first story height is greater than that of the upper stories, it is necessary to increase the approximate column moments materially to equal the accurate moments in the first story. This should be done in all of the above approximate methods.

In the stories at the top of the buildings the column moments for all approximate methods should be increased so

far as per cent of error is concerned. Such may not be necessary so far as actual design is concerned since they are all relatively small stresses.

STORY 35

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